Spiral cylindrique avec courbes terminales en arc de cercle

Développement excentrique et anisochronisme en position horizontale

Déformations planes

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-9}$

Elinvar $\quad \rho_s = 8 \times 10^3\ kg \cdot m^{-3} \quad E = 1.7 \times 10^{11}\,Pa \quad G = 6.538 \times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s \cdot 360 \cdot deg \quad \psi_0 = 3.654 \times 10^3\,deg \quad L := R_0 \cdot \psi_0 \quad L = 318.872\,mm$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0 \cdot (\alpha - \pi) \quad x_{0s}(\alpha) := R_0 \cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0 \cdot sin(\alpha)$

Courbe terminale externe $\quad \beta := 121 \cdot deg \quad \beta_0 := racine\left[\beta \cdot \left(\sqrt{2} \cdot sin(\beta) - 1\right) + sin(\beta) \cdot cos(\beta), \beta\right] \quad \beta_0 = 121.21\,deg$

$$\alpha_A := \pi \quad r_t := \frac{R_0}{\sqrt{2} \cdot sin(\beta_0)} \quad x_{0t}(\alpha_t) := -R_0 + r_t \cdot (1 + cos(\alpha_t)) \quad y_{0t}(\alpha_t) := r_t \cdot sin(\alpha_t) \quad l_t := r_t \cdot 2 \cdot \beta_0$$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0 + \pi, 2 \cdot \pi) \quad \alpha_B = 234\,deg$

$$x_{0t'}(\alpha_{t'}) := \left[R_0 + r_t \cdot \left(-1 + cos(\alpha_{t'})\right)\right] cos(\alpha_B) - r_t \cdot sin(\alpha_{t'}) \cdot sin(\alpha_B)$$

$$y_{0t'}(\alpha_{t'}) := \left[R_0 + r_t \cdot \left(-1 + cos(\alpha_{t'})\right)\right] sin(\alpha_B) + r_t \cdot sin(\alpha_{t'}) \cdot cos(\alpha_B) \qquad L_t := 2 \cdot l_t + L$$

Position du piton $\quad \alpha_{tP} := \pi - 2 \cdot \beta_0 \quad \alpha_{tP} = -62.426\,deg \quad x_P := x_{0t}(\alpha_{tP}) \quad y_P := y_{0t}(\alpha_{tP})$

$z_P := x_P + i \cdot y_P \quad r_P := |z_P| \quad r_P = 3.811\,mm \quad arg(z_P) = -74.047\,deg$

Position du point d'attache à la virole $\quad r_V := r_P \quad \alpha_V(\theta) := Atan\left(x_{0t'}(2 \cdot \beta_0), y_{0t'}(2 \cdot \beta_0)\right) + \theta \quad \alpha_V(0) = 128.047\,deg$

$x_V(\theta) := r_V \cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V \cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 := 270 \cdot deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha) \quad W_{f3} := W_{f\_rect}(ép, ha) \quad \sigma_{max} := \frac{E \cdot I_{33}}{L \cdot W_{f3}} \cdot \theta_0 \quad \sigma_{max} = 113.054 \frac{N}{mm^2}$$

Centres de masse

Partie cylindrique $\quad z_{0s}(\alpha) := x_{0s}(\alpha) + i \cdot y_{0s}(\alpha)$

$$\zeta_{0s} := \frac{R_0}{L} \cdot \int_{\pi}^{\psi_0 + \pi} z_{0s}(\alpha)\, d\alpha \quad \xi_{0s} := Re(\zeta_{0s}) \quad \eta_{0s} := Im(\zeta_{0s}) \quad \xi_{0s} = -0.063\,mm \quad \eta_{0s} = -0.032\,mm$$

Courbe terminale externe $\quad z_{0t}(\alpha_t) := x_{0t}(\alpha_t) + i \cdot y_{0t}(\alpha_t)$

$$\zeta_{0t} := \frac{r_t}{l_t} \cdot \int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t)\, d\alpha_t \quad \xi_{0t} := Re(\zeta_{0t}) \quad \eta_{0t} := Im(\zeta_{0t}) \quad \xi_{0t} = 0\,mm$$

$$\eta_{0t} = 1.429\,mm$$

Courbe terminale interne $\qquad z_{0t'}(\alpha_{t'}) := x_{0t'}(\alpha_{t'}) + i \cdot y_{0t'}(\alpha_{t'})$

$$\zeta_{0t'} := \frac{r_t}{l_t} \cdot \int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\, d\alpha_{t'} \qquad \xi_{0t'} := Re(\zeta_{0t'}) \qquad \eta_{0t'} := Im(\zeta_{0t'}) \qquad \xi_{0t'} = 1.156\,mm \qquad \eta_{0t'} = -0.84\,mm$$

Centre de masse du spiral $\qquad \zeta_s := \frac{1}{L_t} \cdot (L \cdot \zeta_{0s} + l_t \cdot \zeta_{0t} + l_t \cdot \zeta_{0t'}) \qquad \zeta_s = 0\,mm$

Première approximation de la déformée du spiral

Courbe terminale externe

$$\varphi_{0t}(\alpha_t) := \alpha_t + \frac{\pi}{2} \qquad z_{1t}(\theta, \alpha_t) := z_P + r_t \cdot \int_{\alpha_{tP}}^{\alpha_t} i \cdot exp(i \cdot \alpha'_t) \cdot exp\left[i \cdot \theta \cdot \frac{r_t}{L_t} \cdot (\alpha'_t - \alpha_{tP})\right] d\alpha'_t$$

$$z_{1t}(\theta, \alpha_t) := z_P + \frac{r_t \cdot L_t}{L_t + \theta \cdot r_t} \cdot \left(exp\left(-i \cdot \frac{-\alpha_t \cdot L_t - \theta \cdot r_t \cdot \alpha_t + \theta \cdot r_t \cdot \alpha_{tP}}{L_t}\right) - exp(i \cdot \alpha_{tP})\right)$$

Partie cylindrique

$$\varphi_0(\alpha) := \alpha + \frac{\pi}{2} \qquad \Delta z_{1s}(\theta, \alpha) := R_0 \cdot \int_{\pi}^{\alpha} i \cdot exp(i \cdot \alpha') \cdot exp\left(i \cdot \theta \cdot R_0 \cdot \frac{\alpha' - \pi}{L_t}\right) d\alpha'$$

$$\Delta z_{1s}(\theta, \alpha) := \frac{R_0 \cdot L_t}{L_t + \theta \cdot R_0} \cdot \left(exp\left(-i \cdot \frac{-\alpha \cdot L_t - \theta \cdot R_0 \cdot \alpha + \theta \cdot R_0 \cdot \pi}{L_t}\right) + 1\right) \qquad z_{1A}(\theta) := z_{1t}(\theta, \pi)$$

$$\Delta\varphi_{1A}(\theta) := \theta \cdot \frac{l_t}{L_t} \qquad \Delta\varphi_{1A}(\theta_0) = 13.346\,deg \qquad z_{1s}(\theta, \alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta, \alpha) \cdot e^{i \cdot \Delta\varphi_{1A}(\theta)}$$

Courbe terminale interne

$$\Delta z_{1t'}(\theta, \alpha_{t'}) := r_t \cdot \int_0^{\alpha_{t'}} i \cdot exp(i \cdot \alpha'_{t'}) \cdot exp\left(i \cdot \theta \cdot \frac{r_t}{L_t} \cdot \alpha'_{t'}\right) d\alpha'_{t'}$$

$$\Delta z_{1t'}(\theta, \alpha_{t'}) := \frac{r_t \cdot L_t}{\theta \cdot r_t + L_t} \cdot \left(exp\left(i \cdot \alpha_{t'} \cdot \frac{\theta \cdot r_t + L_t}{L_t}\right) - 1\right) \qquad z_{1B}(\theta) := z_{1s}(\theta, \psi_0 + \pi) \qquad \alpha_B = 234\,deg$$

$$\alpha_{1B}(\theta) := Atan(Re(z_{1B}(\theta)), Im(z_{1B}(\theta))) \qquad z_{1t'}(\theta, \alpha_{t'}) := z_{1B}(\theta) + \Delta z_{1t'}(\theta, \alpha_{t'}) \cdot e^{i \cdot \alpha_{1B}(\theta)}$$

Graphe de la déformation

Forme naturelle

$$n_t := 101 \qquad j := 0 .. n_t - 1 \qquad \Delta\alpha_t := \frac{2 \cdot \beta_0}{n_t - 1} \qquad \alpha_{t_j} := \pi - 2 \cdot \beta_0 + j \cdot \Delta\alpha_t \qquad x_{t_j} := x_{0t}(\alpha_{t_j}) \qquad y_{t_j} := y_{0t}(\alpha_{t_j})$$

$$n := 20 \cdot partentière(n_s) + 1 \qquad i := 0 .. n - 1 \qquad \Delta\alpha := \frac{\psi_0}{n - 1} \qquad \alpha_i := \pi + i \cdot \Delta\alpha$$

$$\alpha_{t'_j} := j \cdot \Delta\alpha_t \qquad x_{t'_j} := x_{0t'}(\alpha_{t'_j}) \qquad y_{t'_j} := y_{0t'}(\alpha_{t'_j})$$

$$x_{s_i} := x_{0s}(\alpha_i) \qquad y_{s_i} := y_{0s}(\alpha_i) \qquad x_0 := pile(x_t, x_s) \qquad y_0 := pile(y_t, y_s) \qquad x_0 := pile(x_0, x_{t'}) \qquad y_0 := pile(y_0, y_{t'})$$

$$r_0 := \overrightarrow{\sqrt{x_0^2 + y_0^2}} \qquad \beta_s := \overrightarrow{Atan(x_0, y_0)}$$

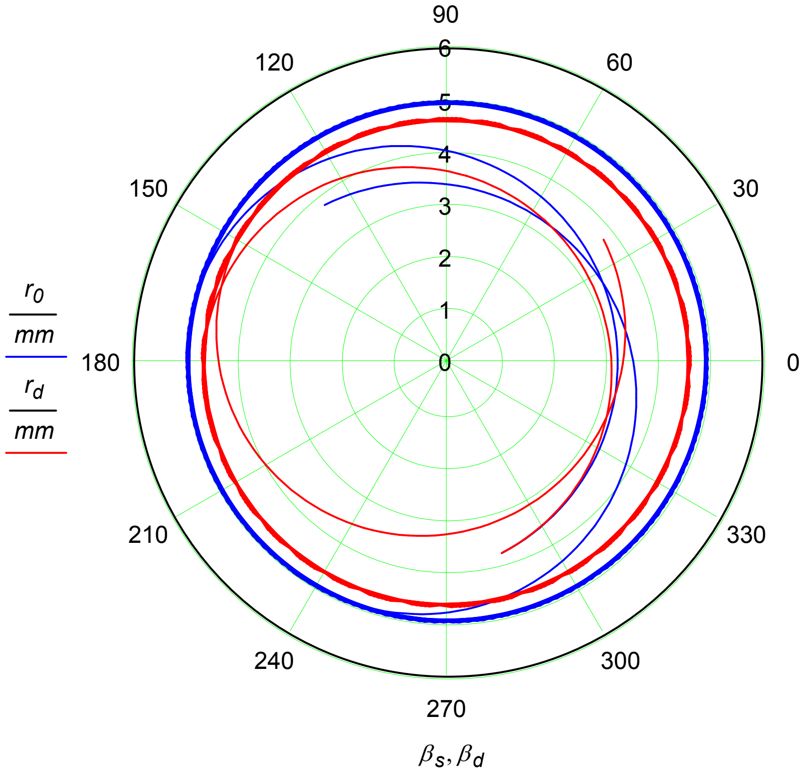
Déformée

$z_{td} := \overrightarrow{z_{1t}(\theta_0, \alpha_t)}$ $\quad z_{sd} := \overrightarrow{z_{1s}(\theta_0, \alpha)}$ $\quad z_d := pile(z_{td}, z_{sd})$

$z_{t'd} := \overrightarrow{z_{1t'}(\theta_0, \alpha_{t'})}$ $\quad z_d := pile(z_d, z_{t'd})$ $\quad n_{pt} := dernier(z_d)$

$x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{|z_d|}$ $\quad r_{d_{npt}} = 3.803\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 285.953\,deg$ $\quad \beta_{d_{npt}} = 38.025\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 38.047\,deg$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 3.811\,mm$

$r_V = 3.811\,mm$

$\alpha_V(0) = 128.047\,deg$

$x_V(\theta_0) = 3.001\,mm$

$y_V(\theta_0) = 2.349\,mm$

$\Delta x_V := x_{d_{npt}} - x_V(\theta_0)$

$\Delta x_V = -5.583 \times 10^{-3}\,mm$

$\Delta y_V := y_{d_{npt}} - y_V(\theta_0)$

$\Delta y_V = -6.264 \times 10^{-3}\,mm$

Déplacement de la virole libre

Contribution de la partie cylindrique du spiral

$s_s(\alpha) := R_0\cdot(\alpha - \pi) + l_t$ $\quad f_s(\theta, \alpha) := \mathrm{i}\cdot\theta\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$\Delta_s(\theta) := \frac{R_0}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot f_s(\theta, \alpha)\,d\alpha$ $\quad \Delta_s(\theta_0) = 0.1 + 0.309\mathrm{i}\,mm$

Approximation $\quad OA := R_0\cdot e^{\mathrm{i}\cdot\pi}$ $\quad OB := R_0\cdot e^{\mathrm{i}\cdot(\pi+\psi_0)}$ $\quad f'_s(\theta, \alpha) := \frac{-\theta^2}{L_t}\cdot R_0\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\left[\left(\mathrm{i}\cdot f_s(\theta, \pi) - f'_s(\theta, \pi)\right)\cdot OA + \left(-\mathrm{i}\cdot f_s(\theta, \pi+\psi_0) + f'_s(\theta, \pi+\psi_0)\right)\cdot OB\right]$$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\theta\cdot e^{\mathrm{i}\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(-OA + e^{\mathrm{i}\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right) + \frac{R_0^2}{L_t^2}\cdot\theta^2\cdot e^{\mathrm{i}\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(OA - e^{\mathrm{i}\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right)$$

$\Delta_{as}(\theta_0) = 0.1 + 0.308\mathrm{i}\,mm$

Contribution de la courbe terminale externe

$s_t(\alpha_t) := r_t \cdot (\alpha_t - \alpha_{tP})$

$$\boldsymbol{\Delta_t}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_t \cdot \int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t)\cdot exp\left(i\cdot\theta\cdot\frac{s_t(\alpha_t)}{L_t}\right) d\alpha_t \qquad \boldsymbol{\Delta_t}(\theta_0) = -0.286 - 0.059i\,mm$$

Approximation

$$f_t(\theta,\alpha_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_t(\alpha_t)}{L_t}\right) \qquad f'_t(\theta,\alpha_t) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_t(\alpha_t)}{L_t}\right)$$

$$\boldsymbol{Og_1} := \frac{r_t}{l_t}\cdot\int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t)\, d\alpha_t \qquad \boldsymbol{Og_2} := \frac{2\cdot r_t^2}{l_t^2}\cdot\int_{\alpha_{tP}}^{\pi} \alpha_t\cdot z_{0t}(\alpha_t)\, d\alpha_t$$

$\boldsymbol{Og_1} = 1.429i\,mm$ $\qquad$ $\boldsymbol{Og_2} = -1.542 + 1.627i\,mm$

$$\boldsymbol{\Delta_{at}}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_t(\theta,0)\cdot\boldsymbol{Og_1} + f'_t(\theta,0)\cdot\frac{l_t^2}{2\cdot r_t}\cdot\boldsymbol{Og_2}\right) \qquad \boldsymbol{\Delta_{at}}(\theta_0) = -0.288 - 0.062i\,mm$$

Contribution de la courbe terminale interne

$s_{t'}(\alpha_{t'}) := r_t\cdot\alpha_{t'} + L + l_t$ $\qquad$ $\alpha_B = 234\,deg$

$$\boldsymbol{\Delta_t'}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}\right)\cdot r_t\, d\alpha_{t'} \qquad \boldsymbol{\Delta_t'}(\theta_0) = 0.197 - 0.216i\,mm$$

Approximation

$$f_{t'}(\theta,\alpha_{t'}) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}\right) \qquad f'_{t'}(\theta,\alpha_{t'}) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}\right)$$

$$f_a(\theta,\alpha_{t'}) := f_{t'}(\theta,0) + \alpha_{t'}\cdot f'_{t'}(\theta,0) \qquad f_a\left(\theta_0,\frac{\pi}{3}\right) = 4.648 - 0.823i \qquad f_{t'}\left(\theta_0,\frac{\pi}{3}\right) = 4.64 - 0.822i$$

$$f_a(\theta,\alpha_{t'}) := f_{t'}(\theta,0) + \frac{1}{r_t}\cdot s_t(\alpha_{t'})\cdot f'_{t'}(\theta,0) \qquad f_a\left(\theta_0,\frac{\pi}{3}\right) = 4.713 - 0.548i$$

$$\boldsymbol{\Delta_{at}'}(\theta) := \frac{1}{L_t}\cdot\int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\cdot\left[f_{t'}(\theta,0) + \frac{1}{r_t}\cdot(r_t\cdot\alpha_{t'})\cdot f'_{t'}(\theta,0)\right]\cdot r_t\, d\alpha_{t'} \qquad \boldsymbol{\Delta_{at}'}(\theta_0) = 0.2 - 0.212i\,mm$$

$$\boldsymbol{\Delta_{at}'}(\theta) := \frac{1}{L_t}\cdot\left[r_t\cdot f_{t'}(\theta,0)\cdot\int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\, d\alpha_{t'} + f'_{t'}(\theta,0)\cdot\int_0^{2\cdot\beta_0}(r_t\cdot\alpha_{t'})\cdot z_{0t'}(\alpha_{t'})\, d\alpha_{t'}\right]$$

$$\boldsymbol{Og'_1} := \frac{r_t}{l_t}\cdot\int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\, d\alpha_{t'} \qquad \boldsymbol{Og'_1} = 1.156 - 0.84i\,mm \qquad \xi_{0t'} = 1.156\,mm \qquad \eta_{0t'} = -0.84\,mm$$

$$\boldsymbol{Og'_2} := \frac{2\cdot r_t}{l_t^2}\cdot\int_0^{2\cdot\beta_0}(r_t\cdot\alpha_{t'})\cdot z_{0t'}(\alpha_{t'})\, d\alpha_{t'} \qquad \boldsymbol{Og'_2} = 1.307 + 0.956i\,mm$$

$$\boldsymbol{\Delta_{at}'}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'}(\theta,0)\cdot\boldsymbol{Og'_1} + f'_{t'}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_t}\cdot\boldsymbol{Og'_2}\right) \qquad \boldsymbol{\Delta_{at}'}(\theta_0) = 0.2 - 0.212i\,mm$$

Contribution du spiral entier

$\Delta_1(\theta) := \Delta_t(\theta) + \Delta_s(\theta) + \Delta_{t'}(\theta)$ $\quad\quad \Delta_1(\theta_0) = 0.011 + 0.035i\,mm$

$u_1(\theta) := Re(\Delta_1(\theta)) \quad v_1(\theta) := Im(\Delta_1(\theta)) \quad u_1(\theta_0) = 0.011\,mm \quad v_1(\theta_0) = 0.035\,mm$

Approximation

$\Delta_a(\theta) := \Delta_{at}(\theta) + \Delta_{as}(\theta) + \Delta_{at'}(\theta)$ $\quad\quad \Delta_a(\theta_0) = 0.012 + 0.034i\,mm$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_{\alpha_{tP}}^{\pi} x_{0t}(\alpha_t)^2\cdot r_t\,d\alpha_t + \int_0^{2\cdot\beta_0} x_{0t'}(\alpha_{t'})^2\cdot r_t\,d\alpha_{t'}\right) \quad p2_{0s} = 12.039\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} y_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_{\alpha_{tP}}^{\pi} y_{0t}(\alpha_t)^2\cdot r_t\,d\alpha_t + \int_0^{2\cdot\beta_0} y_{0t'}(\alpha_{t'})^2\cdot r_t\,d\alpha_{t'}\right) \quad q2_{0s} = 12.105\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot R_0\,d\alpha + \int_{\alpha_{tP}}^{\pi} x_{0t}(\alpha_t)\cdot y_{0t}(\alpha_t)\cdot r_t\,d\alpha_t + \int_0^{2\cdot\beta_0} x_{0t'}(\alpha_{t'})\cdot y_{0t'}(\alpha_{t'})\cdot r_t\,d\alpha_{t'}\right)$$

$$k_{0s} = -0.046\,mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s}\end{pmatrix} \quad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \quad \mathbf{R'}(\theta_0) = \begin{pmatrix} 9.979\times10^{-6} \\ 3.121\times10^{-5}\end{pmatrix} N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 3.277\times10^{-5}\,N$$

Approximations

$$\sigma2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0} \left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\,d\alpha + \int_{\alpha_{tP}}^{\pi} \left(\left|z_{0t}(\alpha_t)\right|\right)^2\cdot r_t\,d\alpha_t + \int_0^{2\cdot\beta_0} \left(\left|z_{0t'}(\alpha_{t'})\right|\right)^2\cdot r_t\,d\alpha_{t'}\right]$$

$$\sigma2 = 24.144\,mm^2 \quad R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \quad R'(\theta_0) = \begin{pmatrix} 1.012\times10^{-5} \\ 3.116\times10^{-5}\end{pmatrix} N \quad \left|R'(\theta_0)\right| = 3.276\times10^{-5}\,N$$

Deuxième approximation de la déformée du spiral

$R'_x(\theta) := \mathbf{R'}(\theta)_0 \quad\quad R'_y(\theta) := \mathbf{R'}(\theta)_1$

$x_{1t}(\theta,\alpha_t) := Re(z_{1t}(\theta,\alpha_t)) \quad\quad y_{1t}(\theta,\alpha_t) := Im(z_{1t}(\theta,\alpha_t))$

$x_{1s}(\theta,\alpha) := Re(z_{1s}(\theta,\alpha)) \quad\quad y_{1s}(\theta,\alpha) := Im(z_{1s}(\theta,\alpha))$

$x_{1t'}(\theta,\alpha_{t'}) := Re(z_{1t'}(\theta,\alpha_{t'})) \quad\quad y_{1t'}(\theta,\alpha_{t'}) := Im(z_{1t'}(\theta,\alpha_{t'}))$

$$s\xi_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}(\theta,\alpha'_t)\cdot r_t\,d\alpha'_t \quad\quad s\xi_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')\cdot R_0\,d\alpha'$$

$$s\xi_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} x_{1t'}(\theta,\alpha'_{t'})\cdot r_t\,d\alpha'_{t'} \quad\quad \xi_1(\theta) := \frac{1}{L_t}\cdot\left(s\xi_{1t}(\theta,\pi) + s\xi_{1s}(\theta,\psi_0+\pi) + s\xi_{1t'}(\theta,2\cdot\beta_0)\right)$$

$$s\eta_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} y_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad s\eta_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} y_{1s}(\theta,\alpha')\cdot R_0\, d\alpha'$$

$$s\eta_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} y_{1t'}(\theta,\alpha'_{t'})\cdot r_t\, d\alpha'_{t'} \qquad \eta_1(\theta) := \frac{1}{L_t}\cdot\left(s\eta_{1t}(\theta,\pi) + s\eta_{1s}(\theta,\psi_0+\pi) + s\eta_{1t'}(\theta,2\cdot\beta_0)\right)$$

$$sp2_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sp2_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')^2\cdot R_0\, d\alpha'$$

$$sp2_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} x_{1t'}(\theta,\alpha'_{t'})^2\cdot r_t\, d\alpha'_{t'}$$

$$sq2_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} y_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sq2_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} y_{1s}(\theta,\alpha')^2\cdot R_0\, d\alpha'$$

$$sq2_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} y_{1t'}(\theta,\alpha'_{t'})^2\cdot r_t\, d\alpha'_{t'}$$

$$sk_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}(\theta,\alpha'_t)\cdot y_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad sk_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')\cdot y_{1s}(\theta,\alpha')\cdot R_0\, d\alpha'$$

$$sk_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} x_{1t'}(\theta,\alpha'_{t'})\cdot y_{1t'}(\theta,\alpha'_{t'})\cdot r_t\, d\alpha'_{t'}$$

$$\mathbf{S_t}(\theta,\alpha_t) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) + sq2_{1t}(\theta,\alpha_t) & \left(y_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t)\right) \\ x_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t) & -x_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) + sp2_{1t}(\theta,\alpha_t) \end{bmatrix}$$

$$\mathbf{S_s}(\theta,\alpha) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) + sq2_{1s}(\theta,\alpha) & \left(y_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha)\right) \\ x_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha) & -x_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) + sp2_{1s}(\theta,\alpha) \end{bmatrix}$$

$$\mathbf{S_{t'}}(\theta,\alpha_{t'}) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1t'}(\theta,\alpha_{t'})\cdot s\eta_{1t'}(\theta,\alpha_{t'}) + sq2_{1t'}(\theta,\alpha_{t'}) & \left(y_{1t'}(\theta,\alpha_{t'})\cdot s\xi_{1t'}(\theta,\alpha_{t'}) - sk_{1t'}(\theta,\alpha_{t'})\right) \\ x_{1t'}(\theta,\alpha_{t'})\cdot s\eta_{1t'}(\theta,\alpha_{t'}) - sk_{1t'}(\theta,\alpha_{t'}) & -x_{1t'}(\theta,\alpha_{t'})\cdot s\xi_{1t'}(\theta,\alpha_{t'}) + sp2_{1t'}(\theta,\alpha_{t'}) \end{bmatrix}$$

$$\boldsymbol{\Delta z_t}(\theta,\alpha_t) := \mathbf{S_t}(\theta,\alpha_t)\cdot\mathbf{R'}(\theta) \qquad \boldsymbol{\Delta z_s}(\theta,\alpha) := \mathbf{S_s}(\theta,\alpha)\cdot\mathbf{R'}(\theta) \qquad \boldsymbol{\Delta z_{t'}}(\theta,\alpha_{t'}) := \mathbf{S_{t'}}(\theta,\alpha_{t'})\cdot\mathbf{R'}(\theta)$$

$$\Delta z_{1t}(\theta,\alpha_t) := \boldsymbol{\Delta z_t}(\theta,\alpha_t)_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_t}(\theta,\alpha_t)_1 \qquad z_{2t}(\theta,\alpha_t) := z_{1t}(\theta,\alpha_t) + \Delta z_{1t}(\theta,\alpha_t)$$

$$\Delta z_{1s}(\theta,\alpha) := \boldsymbol{\Delta z_s}(\theta,\alpha)_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_s}(\theta,\alpha)_1 \qquad z_{2s}(\theta,\alpha) := z_{1s}(\theta,\alpha) + \Delta z_{1s}(\theta,\alpha)$$

$$\Delta z_{1t'}(\theta,\alpha_{t'}) := \boldsymbol{\Delta z_{t'}}(\theta,\alpha_{t'})_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_{t'}}(\theta,\alpha_{t'})_1 \qquad z_{2t'}(\theta,\alpha_{t'}) := z_{1t'}(\theta,\alpha_{t'}) + \Delta z_{1t'}(\theta,\alpha_{t'})$$

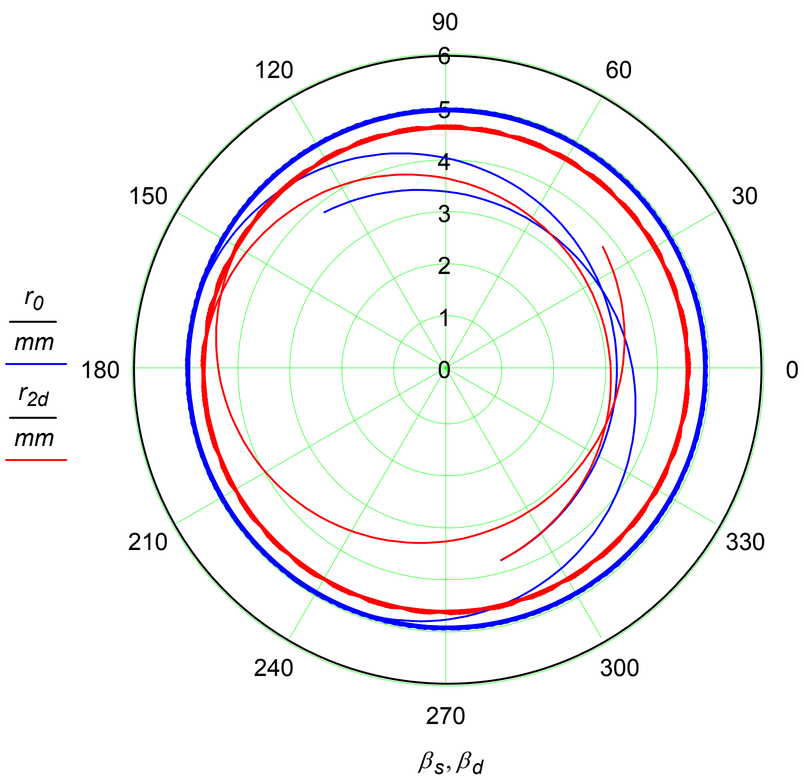
Graphe de la déformation (2ème approximation)

$$z_{2d} := \overrightarrow{z_{2t}(\theta_0,\alpha_t)} \qquad z_{2sd} := \overrightarrow{z_{2s}(\theta_0,\alpha)} \qquad z_{2d} := pile(z_{2d}, z_{sd})$$

$$z_{2t'd} := \overrightarrow{z_{2t'}(\theta_0,\alpha_{t'})} \qquad z_{2d} := pile(z_{2d}, z_{2t'd}) \qquad n_{pt} := dernier(z_d)$$

$$x_{2d} := Re(z_{2d}) \qquad y_{2d} := Im(z_{2d}) \qquad r_{2d} := \overrightarrow{|z_{2d}|} \qquad r_{2d_{npt}} = 3.804\,mm$$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 285.953\,deg$ $\quad \beta_{d_{npt}} = 38.025\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 38.047\,deg$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 3.811\,mm$

$r_V = 3.811\,mm$

$\alpha_V(0) = 128.047\,deg$

$x_V(\theta_0) = 3.001\,mm$

$y_V(\theta_0) = 2.349\,mm$

$\Delta x_V := x_{2d_{npt}} - x_V(\theta_0)$

$\Delta x_V = -5.468 \times 10^{-3}\,mm$

$\Delta y_V := y_{2d_{npt}} - y_V(\theta_0)$

$\Delta y_V = -4.455 \times 10^{-3}\,mm$

Perturbation de période - spiral non déformé en position de repos

$X(\theta) := \frac{\left(\left|\mathbf{\Delta_1}(\theta)\right|\right)^2}{\sigma 2}$ $\quad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$ $\quad Delta(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi$

$\mu(\theta_0) := -86400\cdot Delta(\theta_0)$ $\quad \boxed{\mu(\theta_0) = 0.155}$ $\quad \boxed{\mu(180\cdot deg) = 0.251}$

$X(\theta) := \frac{\left(\left|\mathbf{\Delta_a}(\theta)\right|\right)^2}{\sigma 2}$ $\quad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$ $\quad \delta_a(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma(\theta_0\cdot cos(\varphi))\cdot cos(\varphi)\,d\varphi$

$\mu_a(\theta_0) := -86400\cdot\delta_a(\theta_0)$ $\quad \boxed{\mu_a(\theta_0) = 0.207}$ $\quad \boxed{\mu_a(180\cdot deg) = 0.255}$

$\theta_m := 180\cdot deg, 190\cdot deg\,..\,360\cdot deg$